Chiral Torsion and its quantum effects

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Future Trends in Gravitational Physics

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- $\nabla_\mu \gamma_\nu = 0 \quad \Rightarrow \quad \nabla_\mu e^a_\nu = 0$

  $\Rightarrow \quad e^\lambda_a \partial_\mu e^a_\nu + A^a_{\mu b} e^b_\nu e^\lambda_a - \Gamma^\lambda_{\mu\nu} = 0$ “tetrad postulate”

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$$S = \int |e| d^4x \left[\frac{1}{2\kappa} F_{\mu\nu}^{ab} e_a^\mu e_b^\nu + \frac{i}{2} \left(\bar{\psi} \not{D} \psi - (\bar{\psi} \not{D} \psi)^\dagger \right) + im\bar{\psi}\psi \right]$$

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- Work with one species for the moment

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  - Dynamically generated, completely antisymmetric (geodesic eq. unaffected)

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- Dirac Eq. in curved spacetime is naturally nonlinear (old stuff)

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- However, something else should be taken into account

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- All the equations are exactly the same as before; so what did we gain?

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- So geodesic equation is unaffected – all particles fall at the same rate

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- Effective four-fermion interaction

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Collaborators

Work done with S. Chakrabarty: 1907.02341, 1904.06036
Also I. Ghose, R. Barik, A. Chakraborty (In preparation)

Thank You