An Introduction to the problem of Quark confinement

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Introduction to particle physics

- Fundamental particles: The finest constituents of matter.
- Fundamental interactions: An interactions between fundamental particles is fundamental when it can not be described in terms of other interactions.
Fundamental Particles

- Matter particles
- Gauge particles
Fundamental Particles

- Matter particles
- Gauge particles

Force mediator
Fundamental particles

Matter particles
- Quarks (6)
- Neutrinos (3)
- Electrons (3)

Gauge particles (Force mediator)
Fundamental particles

Matter particles
- Quarks (6)
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Gauge particles (Force mediator)
- Gluons (Strong interaction)
- Electroweak
- Gravitons
Fundamental particles

Matter particles
- Quarks(6)
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Gauge particles (Force mediator)
- Gluons (Strong interaction)
- Electroweak
  - W, Z bosons
- Gravitons
  - Photons (EM)
Quarks

- Quarks are only fundamental particles which have all fundamental interactions
- u, d, c, s, t, b
- Electric charges: $q_d = -\frac{1}{3}$, $q_u = +\frac{2}{3}$
- Three color charges: Red, Blue, Green.
- Mesons are consists of one quark- anti quark pairs.
- Protons and neutrons are made of three quarks.
Proton

\[
\text{Proton} = u + 2/3 + u + 2/3 - d - 1/3
\]
Neutron

\[ + \frac{2}{3} \]

\[ d \]

\[ -\frac{1}{3} \]
Problems

1) Observed Electric charges are integer multiple Of the charge of an electron. Why?

2) Free quarks have not been observed. Why?
Regge trajectories
Regge trajectories

\[ J = \alpha' E^2 \]

\( J = \) Angular momentum, \( E = \) The energy in the center of mass frame

\( \alpha' = 1 \text{ (Gev)}^{-2} \)
Suppose that we picture a meson as a straight line of length $L = 2R$, with mass per unit length $\sigma$.

The line rotates about a perpendicular axis through its midpoint.

Endpoints of the line are moving at the speed of light, $v(R) = c = 1$. 
Then for the energy in the rest frame, i.e. the mass, of the spinning stick we have

\[ m = \text{Energy} = 2 \int_0^R \frac{\sigma dr}{\sqrt{1 - v^2(r)}} = 2 \int_0^R \frac{\sigma dr}{\sqrt{1 - r^2/R^2}} = \pi \sigma R, \]
The angular momentum:

\[ J = 2 \int_0^R \frac{\sigma rv(r)dr}{\sqrt{1 - v^2(r)}} = \frac{2}{R} \int_0^R \frac{\sigma r^2 dr}{\sqrt{1 - r^2/R^2}} = \frac{1}{2} \pi \sigma R^2. \]

Comparing the two expressions

\[ J = \frac{1}{2\pi \sigma} m^2 = \alpha' m^2 \]

The constant \( \alpha' \) is known as the “Regge slope.”
• From the data one estimates $\alpha' = 1/(2\sigma\pi) = 0.9 \text{ GeV}^{-2}$, which gives a mass/unit length of the string, or “string tension”, of

$$\sigma \approx 0.18 \text{ GeV}^2 \approx 0.9 \text{ GeV/fm}.$$
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• The spinning stick model is, of course, only a caricature of the real situation.

• In fact the various Regge trajectories do not pass through the origin, and have slightly different slopes.
• To make the model more realistic, one might want to relax the requirement of rigidity, and allow the “stick” to fluctuate in transverse directions.
• This line of thought leads to string theory
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• This line of thought leads to string theory.

• Problem: Since QCD is the theory of quarks and gluons, the question to be answered is how a stick-like or string-like object actually emerges from that theory.
QCD

- QCD is a theory of strong interaction.
- The Gauss Law:

\[ \nabla \cdot \vec{E}^a = \rho_{\text{quark}}^a - gf^{abc} A_k^b E_k^c \]

\[ E^a_k = \text{Color electric field} , \rho^a = \text{Quark density} \]
• Gluon have 8 color charges. These charges are non commutative in nature.

\[ [Q^a, Q^b] = f^{abc} Q^c \]

• Q’s generate a non-Abelian group SU(3) and Action is invariant under this group action on the gauge field \( A^a_\mu \). SU(3) Gauge theory.

• Perturbative calculation of QCD is well established and do not give any answer to the problem.
Asymptotic freedom (Nobel prize in 2004)

- Discovered in 1973 by D. J. Gross, F. Wilczek and H. D. Politzer

- Asymptotic freedom is the property of some gauge theories in which the interaction between the particles, such as quarks, becomes arbitrarily weak at very high energies.

- QCD is asymptotically free
Problem: QCD is the theory of quarks and gluons, the question to be answered is how a stick-like or string-like object actually emerges from that theory.

• One possible answer is via the formation of a color electric flux tube.
• Imagine that the color electric field running between a static quark and anti-quark is squeezed into a cylindrical region.

• Cross-sectional area is nearly constant as quark-anti quark separation $L$ increases.

• In that case the energy stored in the color electric field will grow linearly with quark separation.
Energy = \sigma L \quad \text{with} \quad \sigma = \int d^2 x_\perp \frac{1}{2} \vec{E}^a \cdot \vec{E}^a

where the integration is over a cross-section of the flux tube.

• That means there will be a linearly rising potential energy associated with static sources (the “static quark potential”)

• An infinite energy is required to separate these charges an infinite distance.
ANO strings

- 1957: A.A. Abrikosov: Magnetic fields in type II superconductors are in fact collimated into magnetic flux tubes, known as Abrikosov vortices.
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- 1973: H. B. Nielsen and P. Olesen: Relativistic generalization of Abrikosov magnetic flux tube in Abelian Higgs model
Monopole confinement

- 1974: Y. Nambu: Proposed a toy model of confinement

- If magnetic monopoles existed in nature, and a monopole-anti monopole pair were placed in a type II superconductor, the monopoles would be connected by a magnetic flux tube, and energy stored in the magnetic field would grow linearly with monopole separation.
Dual superconductor

- Dual confinement: Magnetic charge confined by a magnetic flux tube in a condensate of electrically charged objects (Cooper pairs).
Dual superconductor

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- 1976: This example led ’t Hooft and Mandelstam to suggest that the QCD vacuum is a “dual superconductor”.

Dual superconductor

- The idea is that color electrically charged objects (quarks) are confined by an electric flux tube in a condensate of magnetically charged objects (magnetic monopoles).
Color magnetic monopoles

- In a SU(N) scalar Gauge theory it is possible to break the symmetry to Abelian $U(1)^{N-1}$ subgroup.

- Magnetic monopoles can then be identified from the Abelian magnetic field associated with the gauge bosons of the Abelian subgroup.
Abelian Gauge

- In QCD there is no scalar field, but ’t Hooft proposed that a composite gluonic operator, transforming like a scalar field in the adjoint representation of the gauge group could also serve the purpose of singling out an Abelian subgroup.
Abelian Gauge

• “If there is a conventional Higgs mechanism, these monopoles are physically isolated particles. if there is no Higgs mechanism, such as in QCD, then the monopoles may form a condensate.

• This situation may be completely analogous to the condensation of electric charges in case of an Abelian Higgs mechanism; we just interchange electric and magnetic fields and charges.” – ‘t Hooft
Dual confinement of color magnetic monopoles
Dual confinement of color magnetic monopoles

- In a SU(2) scalar gauge theory with two scalar field we have showed that Monopole confinement is possible and we have showed that monopoles are attached at the end of the flux tubes.
Monopole confinement
“I have spent most of my career wishing that we had a really good way to quantitatively understand the mass gap in four-dimensional gauge theory. I hope that this problem will be solved one day.” – E. Witten
THANK YOU